Math and Physics Contest «Phystech School 2019»

Instructions and tips:
  - *No calculators allowed.*
  - *Answers should be accompanied by concise derivations and/or reasoning, so we could understand your solutions better.*
  - *You are offered 9 math problems and 5 physics problems of varying difficulty. Don't get upset if some problems seem too hard. Even one solved problem counts! Try to solve as many problems as you can to challenge yourself and maximize useful feedback from the grader. Good luck!*

Math problems

1. (2 points) Alice visited a grocery store in Wonderland. She bought 1 orange and 2 apples for 11 coins. Then Cheshire Cat came in and paid 14 coins for 1 orange and 3 apples. How much is one orange? How much is one apple?

2. (2 points) In a triangle ABC BK is a bisector, $\angle A = 60°$, $\angle KBC = 30°$, $AK = 5$ cm. Find the perimeter of the triangle.

3. (3 points) Each contest participant earning 25 points or more will receive a prize certificate from the Phystech School for one month of one-to-one online training in Math or Physics with our tutor at a discount of

$$\frac{(y + h)(h + y)(y + 1) + yh(y + 2h - 2) + h^3 - (y^2 + h^2)}{0.04(y + h)^3}$$

   percent. Here y is the participant's year of birth, h is their height in inches. What discount will a participant born in 1995, 6 feet 4 inches tall get if he/she earns 30 points? (1 foot = 12 inches.)

4. (2 points) A computer screen with a 4:3 aspect ratio has a diagonal of 20 inches. Calculate the area of the screen.

5. (2 points) A curious boy composed some geometric sequence with a common ratio of 3 and found that the first 5 terms of this sequence add up to 242. Calculate the scale ratio of the sequence.

6. (3 points) Find the cosine of an angle between vectors $\vec{a}$ and $\vec{b}$ drawn below.
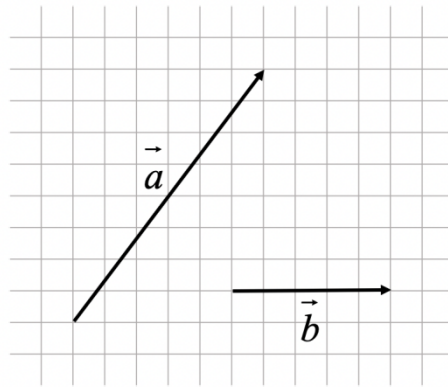

7. (3 points) Evaluate:

$$\frac{\sqrt{3}(\sin 1080° - 2)^3}{1 - 2\sin^2 15°}.$$

8. (3 points) Points B, C, H of a plane α, and point A were chosen in such a way that $AH = 4\sqrt{3}$ is perpendicular to α, AC is a distance from A to BC, $\angle CAH = 30°$, $HB = 2\sqrt{13}$. Find the area of a triangle ABC.

9. (10 points) Each participant of a competition knows exactly 9 other participants. Among any 11 participants there are at least 2 people who know each other. What is the largest possible total number of participants?

Physics problems

1. Andrew and Venus decided to take part in a relay race and called their team "AV". Andrew ran for 20 minutes at 0.3 km/min, and then Venus covered 5 km at $3\frac{1}{3}$ m/s.
   a) (1 point) Who ran for a longer time?
   b) (1 point) Who covered a longer distance?
   c) (1 point) Find the average speed of the team «AV» during the whole relay race.

2. The voltage of the battery is $220\,V$. The resistances are $R_1 = 40\,\Omega$, $R_2 = 120\,\Omega$, $R_3 = 60\,\Omega$, $R_4 = 20\,\Omega$. Find the current flowing out of the battery when key K is:
   a) (1 point) open;
   b) (1 point) closed.
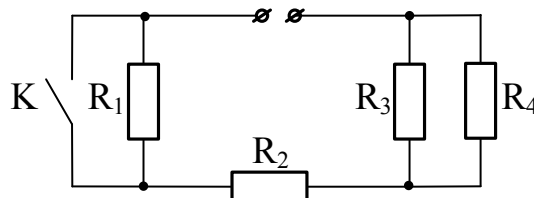
3. (3 points) A helicopter stopped in the air and started to descend along a straight line with an acceleration of $15\ m/s^2$. The engine thrust of this helicopter at descent was $25\sqrt{5}\ kN$, directed horizontally. Find the mass of the helicopter. Neglect air resistance. Assume that the acceleration of free fall is $10\ m/s^2$.

4. (2 points) While natural gas was pumped into a balloon to reach 18 MPa of pressure, its temperature went up to 333 Kelvins. Find the gas pressure after this gas cooled down to 23^0C. Assume that the gas is ideal.

5. (10 points) Victor cut a flat piece of plastic into a circle and rings of equal height (Fig. 1). Then he started to construct "pyramids" from them (Fig. 2 shows the vertical cross-section of the top part of such a pyramid). Plastic is so slippery that it can be moved horizontally with no energy losses. Victor discovered that work A_3 was spent to assemble a 3-level pyramid. What amount of work is needed to increase the number of levels from 40 to 50?

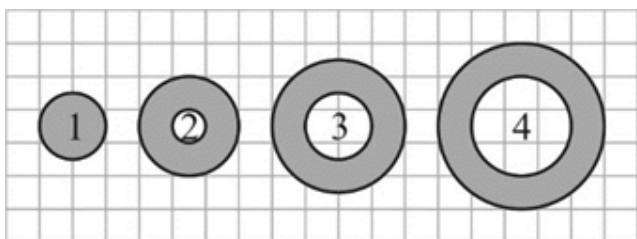*This problem is offered by Ilya M. Gelfgat, a distinguished teacher of Physics.*
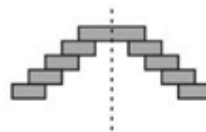

Fig. 1

Fig. 2